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**A review of the doctoral thesis of Jaime de Cabo Martin,  
titled: *Modelling the primordial universe with quantum spacetimes***

**Formal aspects of the thesis**

The thesis of Jaime de Cabo Martin, from now also referred to as the candidate, consists of seven chapters, preceded by an abstract and acknowledgements, and followed by the bibliography. Chapter one is a very concise, three-page description of the motivation for research in quantum cosmology that led to the original results presented in the thesis. Chapter 2 is a technical introduction into quantization methods in quantum cosmology. Chapters 3-6 present the results obtained in the subsequent papers or papers in preparation by the candidate. A one-page-long Chapter 7 describes the conclusions and prospects for the future. The bibliography has 98 items.

**Summary of the formal criteria for a doctoral thesis**

The formal criteria for evaluating a doctoral thesis are specified in the law of the Republic of Poland<sup>1</sup>. According to law, a doctoral thesis should present the candidate's general theoretical knowledge in a scientific discipline and the candidate's ability to carry out their own research. The subject of the doctoral thesis may, in particular, be an original solution of a scientific problem.

**Evaluation of the thesis**

The title of the thesis is very general to the extent that it loosely specifies the content of the thesis. It appears that the title was conceived as a slogan to give an overarching framework for separate projects carried out during the candidate's doctoral studies. This conclusion is supported by the construction of the thesis, in which Chapters 3-6 form separate accounts of different scientific pursuits and do not form a coherent narrative. Put in this way, such an opinion may appear a concern, but a positive take on this observation could be that the thesis reports solutions to not one, but to many scientific problems. Nonetheless, a general theme encompassing all results presented in the thesis would be to seek alternatives to cosmological inflation, both at the level of background and the perturbations.

Chapter 2 gives the theoretical framework of the thesis. From the very beginning it dives into the Hamiltonian formalism for general relativity in the context of cosmology, moves on to a brief

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<sup>1</sup> Ustawa prawo o szkolnictwie wyższym i nauce z dnia 20 lipca 2018 roku z późniejszymi zmianami

exposition of anisotropic models, gives a very short, two-page recount of standard cosmological inflation and then describes at some length coherent states and the quantization methods relying on those states.

I consider Chapter 2 the weakest part of the thesis. It is of a very technical nature, does not provide a conceptual introduction into any of the subjects at the core of reported research, *e.g.* shortcomings of the Standard Cosmological Model, approaches to formulate a quantum theory in the context of early-universe cosmology or different approaches to quantization. Nor does it describe the main hypothesis that drives the research presented in the thesis or even a general problem that the thesis addresses. This is a serious shortcoming in the context of formal criteria mentioned above: the fact that the candidate has general theoretical knowledge in a scientific discipline is not demonstrated in the introductory part of the thesis; instead, it has to be inferred indirectly from chapters presenting the candidate's original research. Fortunately, those chapters abound in the arguments in favor of fulfillment of this criterion.

Chapter 3 presents a simplified cosmological model, a universe filled with a perfect fluid with a constant barotropic parameter. It is shown that upon the quantization of the background and utilizing the generic ordering of the canonical variables generalizing position and momentum – to obtain a repulsive potential for the scale factor and to avoid the initial singularity – two non-equivalent quantization schemes, called the fluid and the conformal parametrizations, can be constructed for the perturbation of the homogeneous and isotropic background. Due to nonlinearity of general relativity resulting in the fact that the transformations between the degrees of freedom used in these two approaches are nonlinear in the quantities that undergo quantization. Hence, there exists an intrinsic theoretical uncertainty about the proper way of formulating quantization scheme in quantum cosmology.

Chapter 3 is written in such a way that the calculations are laid out first and only at the end of this part of the thesis one can find an explanation of the motivation and the significance of those calculations. This bottom-up approach to the narrative does not help in linear reading of the thesis, from generalities to details; on the other hand, section 3.5 is very clearly written and certainly encourages the reader to go back to the earlier parts to appreciate the derivations.

The candidate ends Chapter 3 with a remark that

*there exists an ambiguity in the choice of relevant basic perturbation variables over a quantum background, that might potentially lead to incompatible observational physical predictions.*

The candidate leaves this observation as a cliffhanger, but its quantification and a resolution are immediately provided in Chapter 4. However, one question that is left unaddressed at this stage is whether this ambiguity is maximal or, at least typical, for all different approaches to quantization

that may come about. Also, the strength of the approach presented in Chapter 3 lies in the fact that all gravitational degrees of freedom in this model are quantized on a similar footing. At the same time, the matter content of the universe is consistently described as a perfect fluid, which is a classical notion. In contrast, one of the advantages of inflationary models – and the research presented in the thesis is essentially an attempt to go beyond those models – is that they treat gravity and matter (the inflaton field) on an equal footing. It would be interesting to learn more about the candidate’s perspective in this context.

The cliffhanger in Chapter 3 finds its resolution in Chapter 4, devoted to studying the solutions of the equations of motion for the perturbations. It is found that the fluid and the conformal parametrization yield qualitatively different results for the spectral index of the curvature perturbations. By means of numerical simulations, it was also found that other parametrizations predict the spectral index consistent with that of the conformal parametrization. Additional insight into these findings was obtained by a clever analytical argument showing that the fluid parametrization is a very special case for which the dominant contribution to the curvature perturbation vanishes, so the subdominant modes become important in this non-generic situation. In this way, the ambiguity is effectively resolved.

I find the conclusion about

*the astonishing result that despite the presence of a continuous parameter describing the various possible predictions for the spectral index*

overhyped, as the entire chapter was devoted to extracting the essential features of the evolution of the perturbations and to understanding the parameter dependencies of the predicted power spectra, including the irrelevance of this continuous parameter except a special case. Taken together, Chapter 3 and 4 appear to indicate that, while there may be ambiguities in the proper parametrization of the quantum gravitational degrees of freedom in bouncing cosmological models, the actual predictions for observable quantities are more robust than it may seem at first sight. I view this as an indication that less *academic* (the candidate’s wording) and still predictive models of this type can be built.

In Chapter 5 the two parametrizations introduced in Chapter 3 are further studied for their parameter dependencies. It is found that for the perturbations to match the observed amplitude of the cosmic microwave background fluctuations, the quantum bounce must be extraordinarily strong. Another result regards the outcome for the fluid parametrization, which – based on the parametric dependencies and low particle production – was again deemed unphysical.

Taken together, Chapters 3-5 form an interesting and important narrative, in which a new (fluid) parametrization of the quantum perturbations is found, but the accumulated further research shows that it is just a special point in the parameter landscape leading to unacceptable predictions

for the observed temperature fluctuations of the cosmic microwave background<sup>2</sup>. While introducing a new description and then dismissing it may be understood as a negative result, the considerations presented in this part of the thesis shed much light on the approach to cosmological perturbations in quantum models of spacetime, a new research area with many open questions.

Chapter 6 reports additional results which are loosely related to the rest of the thesis, as a completely new model of the universe – an anisotropic one – is introduced and the analysis is focused on the background evolution, not on the perturbations. By a careful examination, the candidate is able to address a very interesting hypothesis formulated at the beginning of this chapter – whether the presence of the anisotropy can sustain accelerated expansion of the universe, thereby offering an alternative to inflationary models. The answer is negative, which follows from the form of the effective potential for the anisotropy degrees of freedom that – in contrast to the potential of a scalar field employed in inflation – cannot be arbitrarily flat, both in the classical and the quantum model. This is an important conclusion that justifies the inclusion of this chapter to the thesis.

The description of the candidate's contribution to the content of Chapter 4-6 demonstrates the ability to carry out scientific research on his own, thereby fulfilling the second legal requirement for a doctoral degree.

### **Other remarks**

The editing of the thesis is very good, a few isolated typographical errors are barely noticeable. The thesis is written in an elegant way, in a scientific style and with comprehensible syntax. An interesting idea is to finish each of the main chapter with a brief explanation of the results described in each chapter. While I could wish for a text written so clearly that such *a posteriori* elucidations are necessary, this demonstrates the candidate's ability to communicate clearly his complex ideas. In the bibliography, the candidate fell for a fad of referring selectively to original research that forms the foundation of our understanding of the universe (Einstein, references [1-5]). Such care was absent in reference to cosmological inflation, which, at this resolution, could be attributed to:

- A. Starobinsky, *Phys. Lett. B* 91 (1980) 99,
- K. Sato, *Mon. Not. Roy. Astron. Soc.* 195 (1981) 467,
- A. H. Guth, *Phys. Rev. D* 23 (1981) 347,
- A. D. Linde, *Phys. Lett. B* 108 (1982) 389,  
A. Albrecht and P. J. Steinhardt, *Phys. Rev. Lett.* 48 (1982) 1220,
- A. D. Linde, *Phys. Lett. B* 129 (1983) 177.

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<sup>2</sup> The conformal parametrization does not yield satisfactory predictions, either, but the candidate shows convincingly that the fluid parametrization runs into so many difficulties that it should not be viewed as a viable approach.

### Summary and conclusions

Jaime de Cabo Martin's doctoral thesis contains many important results regarding the fully quantum approach to cosmological spacetimes, which shows that he is a mature scientist, capable of solving research problems in this field. In spite of a number of critical remarks presented in my review<sup>3</sup>, **my evaluation of the thesis is fully positive**. Research accomplishments and theoretical knowledge demonstrated in the thesis are more than sufficient to fulfill the criteria for candidates for a doctoral degree. Therefore, I wholeheartedly recommend accepting Jaime de Cabo Martin to further stages of the process of awarding the title of doctor in physical sciences, including the public defense of the thesis.

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<sup>3</sup> Strength of the thesis are succinctly noted, while I feel obliged to elaborate on negative remarks or concerns.