Prof. dr hab. Marek Rogatko
Department of Theoretical Physics
Astrophysics and Gravity Theory Group
Maria Curie-Skłodowska University
pl. M. Curie Skłodowskiej 1
20-031 Lublin, Poland.

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Report on the doctoral thesis by Alice Boldrin "Classical and quantum aspects of perturbations in Primordial Universe".

The doctoral thesis under the title "Classical and quantum aspects of perturbations in Primordial Universe", written by Alice Boldrin, comprises 107 pages. The content is grouped in four chapters, with the supplement 'Discussion and future prospects', bibliography and two appendices. The supervisors of the PhD dissertation were Prof. Przemysław Małkiewicz and Prof. Patrick Peter.

The thesis constitutes the self-contained work, on the results achieved by the Author with the coworkers. As the Author wrote, the main aim of the dissertation was to introduce some useful methods which enable us deepen understanding of gravity quantization.

After the Introduction, in **chapter 1** of the dissertation, the applicable in further investigations mathematical formalism was described. There were treated such subjects as, the basic facts connected with the ADM formalism, Dirac method of treating constrained Hamiltonian and the concept of Dirac observables.

In **chapter 2** the perturbed Hamiltonian up to the n-th order was discussed, then the results were used to achieve the second order perturbations of the metric. The perturbed system was analysed by means of the Dirac method. Using the Dirac observables commutating with the constraints, the Author obtained the final physical Hamiltonian. The spacetime reconstruction,

i.e., finding the value of the lapse and shift functions depending on the gauge choice, were done by taking into account the Poisson brackets of the adequate quantities. Then the method in question was refined via the so-called Kuchar parametrisation of the kinematical phase space. The gauge invariant dynamics of three-surfaces was separated from three-surface deformations caused by linear coordinate transformations. The key point of this approach was to reconstruct the spacetime having in mind gauge-fixing transformations.

The basic ideas of the aforementioned approach was published in A. Boldrin and P. Małkiewicz, *Dirac procedure and the Hamiltonian formalism for cosmological perturbations in a Bianchi I universe*, Class. Quant. Grav. 39, 025005 (2022).

In **chapter 3** the Dirac procedure was implemented to find the physical phase space and physical Hamiltonian for anisotropic cosmological model. The Fermi-Walker-propageted basis was introduce in which the Author found the tensor and vector modes, as well as, the Hamiltonian generated by the choice of the basis in question. Next, after choosing the convenient gauge the physical Hamiltonian was obtained. Among all, it was also revealed that the results were valid in any gauge (when one replaced the physical variables by the adequate Dirac ones). The obtained formalism was additionally applied to the multi-field case, which could be relevant for the description of the very early phase of the Universe.

Chapter 4 is devoted to the problem of quantum fields on quantum background spacetimes. The specific example of a primordial gravitational wave propagating through a bouncing quantum Friedman Universe was elaborated. Among all, it was showed that several dynamical variables like the scale factor, amplitude of gravitational wave, achieved from different internal clocks evolve differently in comparison to a clock-independent description. It happens that the expectation values and mode functions of operators converge to a unique evolution for large classically behaving Universe, as well as, the dynamics (described by different clocks) converges to the classical behaviour at different times.

The very interesting conclusion can be drawn that for large Universes the semi-classical background and quantum perturbation dynamics do not depend on the clock. These concepts were published in: A. Boldrin, P. Małkiewicz, P. Peter, *Time problem in primordial perturbations*, Phy. Rev. D (2023).

At the end of the thesis the Author presents the new directions of the future researches, being generalisation of the presented ones.

The PhD dissertation presented by Alice Boldrin is clearly written, with a good selection of the bibliographical examples. One should also point out the graphical side of the work, which especially in the 1-st chapter, is very helpful in understanding the context. The rest of the plots are taken from her original works.

Summing it up, I have to admit that the Author demonstrated a good command of mathematical aspects of general relativity, especially connected with the modifications of the ADM formalism. In the context of the early Universe recent observations, the results obtained by Alice Boldrin are valuable and interesting.

The achievements presented in the thesis were partially devoted to the investigations and results accomplished with the PhD supervisors and published in prestigious journals like Classical Quantum Gravity and Physical Review D. Therefore my positive evaluation of the doctoral thesis has its confirmation in reviewers' opinions before acceptance of the works for publishing.

My final conclusion is of course positive. The results of investigations presented by Alice Boldrin in her PhD dissertation are the valuable contributions and they meet the criteria prescribed by the law for doctoral dissertation, therefore I recommend Alice Boldrin to the subsequent stages of the PhD procedure, including the public defence.

Moja ocena jest oczywiście pozytywna. Rezultaty przedstawione w pracy doktorskiej pani Alice Boldrin stanowią znaczący wkład do dziedziny jej badań i spełniają kryteria ustawowe stawiane pracom doktorskim, dlatego wnoszę o dopuszczenie pani Alice Boldrin do dalszego postępowania w przewodzie doktorskim.