dr hab. Radosław Ryblewski, prof. IFJ PAN
The Henryk Niewodniczański
Institute of Nuclear Physics
Polish Academy of Sciences
Radzikowskiego 152
31-342 Kraków, Poland

Kraków, November 19, 2021 r.

Review report on the doctoral thesis of Mr. Viktor Svensson titled *Relativistic Hydrodynamics Beyond the Second Order*

The dissertation of Mr. Viktor Svensson titled *Relativistic Hydrodynamics Beyond the Second Order* was prepared under the supervision of dr hab. Michał P. Heller and submitted in fulfilment of the requirements for the degree of Doctor of Physical Sciences at the National Centre for Nuclear Research in Świerk. The dissertation is based on the results of six papers published in high impact peer-reviewed international journals included in the JCR list (3 from *Physical Review D*, 1 from *Journal of High Energy Physics*, 1 from *SciPost Physics*, and 1 from *Physical Review Letters*). All considered articles are collaborative works.

The ultimate goal of the research presented in this thesis is to improve the understanding of the high effectiveness of relativistic fluid dynamics in describing the evolution of strongly interacting matter produced in ultra-relativistic heavy-ion collisions. Given the complex structure of the underlying non-equilibrium quantum field theory, the classical field theory description of hydrodynamics is a dramatic simplification for the problem of modeling the dynamics of such processes. In fact, the theoretical models based on relativistic dissipative hydrodynamics characterize such processes surprisingly well, provided that the fluid-like behavior of the produced matter sets in shortly after the initial nuclear impact and assuming that the dissipative corrections are very small. This observation is particularly remarkable considering that the system under consideration is far from equilibrium at this early stage, and fluid dynamics is traditionally considered a theory applicable to systems close to equilibrium. The seemingly unreasonable effectiveness of fluid dynamics is currently one of the most important puzzles in the field of relativistic nuclear collisions, especially recently since hydrodynamic behavior is also observed in much smaller systems, where the out-of-equilibrium effects are likely to be even more pronounced. **Consequently, the research topics addressed in this thesis are at the forefront of current studies of this field and are of great relevance to ongoing high-energy experiments. Moreover, the universality of hydrodynamics gives these studies a truly interdisciplinary character with potential implications, in particular, for astrophysics, cosmology, and condensed matter physics.**

The thesis has a form of a concise guide through the current status of the discipline, undertaken research topics, ued terminology, and main results which are contained in the series of papers **A-F** that form the basis of this thesis. The thesis is logically structured, clearly written and, apart from a few minor misprints, carefully proofread. The content presented in the thesis is sufficient to get the general perspective on the performed studies. However, to get a complete view of the results obtained and techniques used, the reader is encouraged to familiarise himself with the original papers. In the following, I will briefly describe the main results of the latter.

Hydrodynamics is usually constructed as a perturbative expansion of conserved currents around equilibrium in terms of fundamental hydrodynamic fields and their gradients. The number of terms as well as the transport coefficients in such an expansion grows rapidly with order, which makes the analysis of the behavior at large orders a complicated task. To make the analytical calculations feasible, some symmetries usually have to be imposed on the system, with the Bjorken and Gubser symmetries being the standard examples. In such a case, the equations of motion become ordinary differential equations in which the late-time behavior and the gradient expansion are, in fact, closely related.

Such a symmetry was also employed in papers **A** and **B**, where, for the first time, the transition to hydrodynamics has been explored in a weakly coupled model given by the conformal kinetic theory treated in the relaxation time approximation. Using the resurgence techniques, in paper **A** it was shown that the gradient expansion, as in the case of the previously studied strongly coupled gauge theories, has a vanishing radius of convergence which is associated with the presence of a fast decaying transient (non-hydrodynamic) mode in the model. Interestingly, in addition to the expected purely decaying mode, also other singularities have been observed in the analytic continuation of the Borel transform, which exhibited damped oscillatory behavior and do not correspond to any transient modes in the microscopic theory. It has been shown that these singularities are a manifestation of the analytic properties of the evolution equations in complexified time and therefore proven to be unphysical.

In a similar setup, it was explained in paper **B** how an infinite number of degrees of freedom, which are present in the Boltzmann equation in the relaxation time approximation and correspond to infinite states of the distribution function, manifest themselves in the Borel plane. It has been shown that apparently missing transient effects are in fact of infinite number. However, they all have the same exponential decay determined by the relaxation time, and differ only in different subleading behavior and different oscillation frequencies. In consequence, they are appearing as degenerated in the Borel plane.

The papers **A** and **B** are exploratory not only from the physics point of view, but also in the context of the tools used in the analyses, in particular Borel resummation analysis. Hence, these results extremely valuable for any follow up studies.

The studies presented in paper **C** focus on the physics of hydrodynamic attractors. In simple setups, such as the one of the Bjorken-expanding system described by RTA kinetic theory, they appear as collapses of different solutions for pressure anisotropy to a single trajectory in phase space. Their presence is undestood to arise from the expansion form at early times and due to the exponential decay of the transient modes at late times. However, while the presence of attractors has been confirmed in various simple models, a precise prescription for the selection of observables which may possibly exhibit attractor behavior has been lacking, leaving open the question of attractor existence beyond highly symmetric flows. In the paper **C** a general method to detect attractors was proposed relying on their universal property of the dynamical dimensional reduction of the phase space regions of the theory. The detection of this phenomenon was then realized using the principal component analysis machine learning algorithm and it was shown to work as expected. The presented method is very general as it relies on a universal property of the attractors and is extremely useful for more complex systems. Given recent interest in physics of attractors these result are valuable.

The paper **F** discusses the analysis of the behavior of the hydrodynamic gradient expansion at large orders in the framework of linear response theory. It is shown that the real space hydrodynamic gradient expansion diverges when the initial data have a support in momentum space exceeding a critical value determined by the microscopic theory. Otherwise, it converges. The geometric growth of the hydrodynamic dispersion relation implies the same for the values of the transport coefficients. At the same time, a certain support in the momentum space for the hydrodynamic fields leads to the increse in tensor structures built from repeated derivatives, hence causing geometric growth. Thus, the convergence of the gradient expansion is determined by these two aspects. This is an important insight, because it means that the hydrodynamics itself should not be considered as either convergent or divergent, since it depends on particular solutions.

Although the hydrodynamic expansion may be divergent, keeping only couple first terms of it usually leads to a satisfactory approximation. Apart from some recent first-order formulations, the causality and stability requirements in practical applications require the use of hydrodynamic models resulting from an actual truncation of the series at least above the first order (MIS or BRSSS are examples of such models) or some resummation of the series. The resulting models include both hydrodynamic and non-hydrodynamic modes. It was found that in the case of simple flows, their late-time solutions take the form of a transseries involving both power series and an exponential suppression factor.

The paper **D** presents the study of transseries in BRSSS that go beyond these simple flows. The study is carried out for the case of a linear perturbation around equilibrium, where the shear channel fluctuations in BRSSS theory have the form similar to the relativistic diffusion equation. The introduction of perturbation parameter allows for separation of the transients from the hydrodynamic contributions in the transseries. It is found that the behavior of the transient sector is determined by initial conditions and regions in spacetime.

Finally, inspired by the studies in BRSSS and N = 4 SYM theory, the paper **E** addresses the problem of the convergence radius of the hydrodynamic dispersion relations in RTA kinetic theory. A particularly important result of the paper is the general prescription for finding the radius of convergence of the hydrodynamic modes that results from the Implicit Function Theorem.

Evaluation summary

The dissertation of Mr. Svensson represents a very high quality research. The presentation of the findings is insightful and the results are comprehensively put in proper physics context, demonstrating a solid knowledge of the current state of research in the field. The scientific aims of the dissertation are wide-ranging and ambitious, requiring the candidate to demonstrate a high level of scientific expertise, creativity and independence. The results are of the highest international standard and of great importance for the development of the discipline.

According to the statements of the candidate and his supervisor, the candidate's contribution to every aspect of the preparation of the thesis, including the formulation of ideas, numerical and analytical calculations, discussions and interpretation of results as well as presentation is clearly visible and important, and often exceeds the expectations for a scientist at this career stage.

In summary, I conclude that the doctoral thesis of Mr. Viktor Svensson meets all the criteria required for the doctorate in physics. I therefore unreservedly recommend that the candidate be awarded the doctorate.

In view of the outstanding quality of the research work presented, I propose that Mr. Viktor Svensson be awarded a doctorate with distinction.

dr hab. Radosław Ryblewski